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Supersymmetric orientifolds of Gepner models

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Abstract: Supersymmetric orientifolds of four dimensional Gepner Models are constructed in a systematic way. For all levels of the Gepner model being odd the generic expression for both the A-type and the B-type Klein bottle amplitude is derived. The appearing massless tadpoles are canceled by introducing appropriate boundary states of Recknagel/Schomerus(RS). After determining the Möbius strip amplitude we extract general expressions for the tadpole cancellation conditions. We discuss the issue of chirality for such supersymmetric orientifold models and finally present a couple of examples in detail.

Keywords: D-branes, Superstring Vacua, Conformal Field Models in String Theory.

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Contents

1. Introduction

The last years have seen some encouraging progress in string model building as well as in understanding D-branes in curved space-times. In particular we have learned that intersecting D-brane models [1]–[6] do provide phenomenologically appealing string vacua, where many aspects of the Standard Model, like chirality or family replication, can be achieved quite naturally (for reviews see [7]). For instance, the number of families is given by the topological intersection number between pairs of D-branes wrapping some 3-cycles in the internal space. However, it turned out that quasi realistic supersymmetric models, in particular those with no adjoint scalars, are much more difficult to get, at least in the toroidal orbifold setting one has used so far [5]. Therefore, from this point of view it is necessary to move beyond the simple toroidal orbifold backgrounds.

One step in this direction was performed in [8], where the structure of intersecting D-brane models on general Calabi-Yau spaces was discussed (see also [9, 10]). Unfortunately, for general Calabi-Yau spaces not very much is known about the special lagrangian 3-cycles the D6-branes are supposed to wrap around, hampering any further progress in finding any concrete realization of such intersecting brane models.

However, it is known that certain points in the moduli space of Calabi-Yau compactifications are exactly solvable and that their concrete rational conformal field theory is described by so-called Gepner models [11, 12]. After same more general discussion in [13], triggered by the pioneering work of Recknagel/Schomerus [14] there has been some amount of study on boundary states in these Gepner models [15]–[19]. Geometrically these boundary states correspond to stable supersymmetric D-branes either wrapping special lagrangian 3-cycles in the Calabi-Yau or wrapping holomorphic curves equipped with some coherent sheaves. Part of the data one needs for constructing intersecting D-brane models, like the intersection numbers, are known for these boundary states [16]. Therefore, it is a natural question whether one can actually use them in a concrete way for model building.

Clearly for finally yielding supersymmetric backgrounds it is mandatory to cancel the positive tension of these branes by same negative contributions arising naturally from orientifolds planes. Therefore, we are naturally led to consider orientifolds of Gepner models [20]–[28]. We would like to emphasize that even without these phenomenological ambitions in mind, just from the technical conformal field theory point of view [29]–[35], the construction of fully fledged Gepner model orientifolds is a natural next step to perform.

In fact there has lately been some work on the construction of crosscap states (orientifold planes) in Gepner models respectively in more general $N = 2$ theories [24]–[27]. However, there do exist only very few attempts to really construct fully fledged orientifold models including the derivation of the tadpole cancellation conditions, their solutions and the determination of the resulting massless spectra. After constructing some six dimensional models in [20], the first four dimensional models were studied in [22], where it came as a surprise that the gauge groups could have a much larger rank than initially expected. For instance, it turned out that for the $(3)^5$ Gepner model the gauge group could maximally be $G = \mathrm{SO}(12) \times \mathrm{SO}(20)$. Recently, in [28] many more eight and six-dimensional models were constructed in addition to the example of the four-dimensional $(3)^5$ Gepner model.

So far the construction of models were more like a case by case study with intensive use of computer power. An important step forward would be to derive general results for the Klein bottle and Möbius amplitudes or for the tadpole cancellation conditions. The aim of the current paper is to approach this problem and start a systematic investigation of orientifolds of Gepner model. In the course of the paper we will consider four dimensional B-type and A-type orientifolds of type-IIB on Gepner models. To avoid the subtlety of simple current fixed points and to keep the formulas as simple as possible, we restrict ourselves to the case where all levels of the Gepner model are odd. It would be interesting to generalize this to even levels utilizing the general results of [17, 33, 24].

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We will show that it is indeed possible to derive absolutely general expressions for the Klein bottle amplitude. The appearing massless tadpoles are canceled by introducing RS-boundary states, which pairwise generically break supersymmetry and have non-zero intersection numbers meaning that there are chiral fermions localized on the intersection locus. In this respect they are completely analogous to the D-branes intersecting at angles, which were introduced to get chiral fermions in toroidal orbifold models. However, it will turn out that the requirement of supersymmetry further constrains the model and that relatively supersymmetric B-type boundary states do not admit chiral fermions anymore.

The most sophisticated step is the determination of the Möbius amplitude, which involves extra sign factors in front of the characters. Led by the requirement that the loop and tree channel Möbius amplitudes do respect the GSO projection, we will determine these signs and compute (for simplicity just in the NS sector) general expressions for them. Adding up all different one-loop amplitudes we will extract the massless tadpole cancellation conditions, which turn out to have a very suggestive form. All these amplitudes and conditions can be straightforwardly put on a computer to search for interesting nontrivial solutions. We will discuss a couple of simple B-type examples and leave the more complicated though probably phenomenologically more appealing A-type models for future study.

This paper is organized as follows. In section 2 we review some facts about Gepner models. Section 3 contains some comments about orientifolds of Gepner models in general followed by the computation of the Klein bottle amplitude for the B-type and A-type orientifolds. In section 4 we review some of the important aspects of RS-boundary states including the loop and tree channel annulus amplitudes. The derivation of the Möbius strip amplitudes is the subject of section 5, where for simplicity we restrict ourselves to the explicit computation of the NS sector amplitudes. In section 6 we present the general tadpole cancellation conditions and discuss a couple of examples in section 7. Finally, section 8 contains our conclusions.

2. Review of Gepner models

To set the stage we briefly review some aspects of Gepner models needed in the remainder of this paper. In light cone gauge, the internal sector of a type-II compactification to four dimensions with $N = 2$ supersymmetry is given by an $N = 2$ supersymmetric conformal field theory (SCFT) with central charge $c = 9$. The idea of Gepner was to use tensor products of the well known rational models of the $N = 2$ super Virasoro algebra for this $N = 2$ SCFT [11, 12]. In order to really get space-time supersymmetry one has to invoke a GSO projection, which can be described by a certain simple current in the SCFT.

More concretely, the minimal models are parametrized by the so-called level $k = 1, 2, \ldots$, in term of which the central charge is given by

$$c = \frac{3k}{k+2}\,. \tag{2.1}$$

Since $c < 3$ one has to use tensor products of such minimal models $\bigotimes_{j=1}^{r}(k_j)$ adding up to the required value $c = 9$. Each unitary model has only a finite number of irreducible representations of the $N = 2$ Virasoro algebra, which are labeled by the three integers (l, m, s) in the range

$$l = 0, \dots, k\,, \qquad m = -k-1, -k, \dots, k+2\,, \qquad s = -1, 0, 1, 2 \tag{2.2}$$

with $l + m + s = 0 \bmod 2$. Actually, taking the identification between (l, m, s) and $(k - l, m + k + 2, s + 2)$ into account, the range (2.2) is a double covering of the allowed representations. Since the Cartan subalgebra of the $N = 2$ Virasoro algebra contains the two elements L_0, j_0, each state in the representation carries two quantum numbers, the conformal weight and the U(1) charge. Utilizing the coset construction of each minimal model as $\mathrm{SU}(2)_k \times \mathrm{U}(1)_2/\,\mathrm{U}(1)_{k+2}$, the conformal dimension h and charge q of the highest weight state with label (l, m, s) is given by

$$\begin{aligned} \Delta^l_{m,s} &= \frac{l(l+2) - m^2}{4(k+2)} + \frac{s^2}{8} \quad (\mathrm{mod}\ 1) \\ q^l_{m,s} &= \frac{m}{(k+2)} - \frac{s}{2} \quad (\mathrm{mod}\ 2)\,. \end{aligned} \tag{2.3}$$

Note, that these formulas are only correct modulo one respectively two. The exact conformal dimension h and charge can be read off from (2.3) by first shifting the labels into the standard range $|m - s| \le l$ by using the shift symmetries $m \to m + 2k + 4$,$s \to s + 4$ and the reflection symmetry. Representations with even s belong to the NS-sector while those with odd s belong to the R-sector.

So far we have just given the field content of each $N = 2$ tensor factor. In addition one has the contributions with $c = 3$ from the two uncompactified directions where the two world-sheet fermions $\psi^{2,3}$ generate a $\mathrm{U}(1)_2$ model. This has four irreducible representations labeled by $s_0 = -1, \dots, 2$ with highest weight and charge

$$\Delta_{s_0} = \frac{s_0^2}{8} \quad (\mathrm{mod}\ 1)\,, \qquad q_{s_0} = -\frac{s_0}{2} \quad (\mathrm{mod}\ 2)\,. \tag{2.4}$$

In the superstring construction in order to achieve space-time supersymmetry one has to implement a GSO projection, which in the Gepner case means that one projects onto states with odd overall U(1) charge $Q_{\mathrm{tot}} = q_{s_0} + \sum_{j=1}^{r} q^{l_j}_{m_j,s_j}$. Moreover, to have a good space-time interpretation one has to ensure that in the tensor product only states from the NS respectively the R sectors couple among themselves.

In order to describe these projections in a simple way one introduces the following notations. First one defines some multi-labels

$$\lambda = (l_1, \dots, l_r)\,, \qquad \mu = (s_0; m_1, \dots, m_r; s_1, \dots, s_r) \tag{2.5}$$

and the respective characters

$$\chi^{\lambda}_{\mu}(q) = \chi_{s_0}(q)\, \chi^{l_1}_{m_1,s_1}(q) \dots \chi^{l_r}_{m_r,s_r}(q)\,. \tag{2.6}$$

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Introducing the vectors

$$\beta_0 = (1;1,\ldots,1;1,\ldots,1)\,, \quad \beta_j = (2;0,\ldots,0;0,\ldots,0,\underbrace{2}_{j^{\text{th}}},0,\ldots,0) \tag{2.7}$$

and the following product

$$\begin{aligned} Q_{\text{tot}} &= 2\beta_0 \bullet \mu = -\frac{s_0}{2} - \sum_{j=1}^{r} \frac{s_j}{2} + \sum_{j=1}^{r} \frac{m_j}{k_j+2}\,, \\ \beta_j \bullet \mu &= -\frac{s_0}{2} - \frac{s_j}{2}\,, \end{aligned} \tag{2.8}$$

the projections one has to implement are simply $Q_{\text{tot}} = 2\beta_0 \bullet \mu \in 2\mathbb{Z}+1$ and $\beta_j \bullet \mu \in \mathbb{Z}$ for all $j = 1,\ldots,r$. Gepner has shown that the following GSO projected partition function

$$Z_D(\tau,\overline{\tau}) = \frac{1}{2^r}\frac{(\mathrm{Im}\tau)^{-2}}{|\eta(q)|^4} \sum_{b_0=0}^{K-1} \sum_{b_1,\ldots,b_r=0}^{1} \sum_{\lambda,\mu}^{\beta} (-1)^{s_0}\, \chi_\mu^\lambda(q)\, \chi^\lambda_{\mu+b_0\beta_0+b_1\beta_1+\cdots+b_r\,\beta_r}(\overline{q}) \tag{2.9}$$

is indeed modular invariant and vanishes due to space-time supersymmetry. Here $K = \mathrm{lcm}(4, 2k_j+4)$ and $\sum^\beta$ means that the sum is restricted to those λ and μ in the range (2.2) satisfying $2\beta_0 \bullet \mu \in 2\mathbb{Z}+1$ and $\beta_j \bullet \mu \in \mathbb{Z}$. The factor 2^r due to the field identifications guarantees the correct normalization of the amplitude. In the partition function (2.9) states with odd charge are arranged in orbits under the action of the β vectors. Therefore, the partition function is non-diagonal in the original characters, but for all levels odd it can be written as a diagonal partition function in terms of the orbits which in this case have all equal length $2^r\,K$. These orbits under the β-vectors (2.7) will loosely be called GSO orbits in the following.

Since in the sequel we will make extensive use of the modular S-transformation rules for the characters involved in (2.9), let us also state them here. For the $\mathrm{SU}(2)_k$ Kac-Moody algebra the S-matrix is given by

$$S_{l,l'} = \sqrt{\frac{2}{k+2}}\,\sin(l,l')_k\,, \tag{2.10}$$

where we have used the convention $(l,l')_k = \frac{\pi(l+1)(l'+1)}{k+2}$. For the $N=2$ minimal model one obtains for the modular S-matrix

$$\begin{aligned} S^{\mathrm{U}(1)_2}_{s_0,s_0'} &= \frac{1}{2} e^{-i\pi\frac{s_0 s_0'}{2}}\,, \\ \mathcal{S}_{(l,m,s),(l',m',s')} &= \frac{1}{2\sqrt{2k+4}}\, S_{l,l'}\, e^{i\pi\frac{m\,m'}{k+2}}\, e^{-i\pi\frac{s\,s'}{2}}\,. \end{aligned} \tag{2.11}$$

Note, that in the latter expression there is a difference in the normalization of the S-matrix of a factor $\sqrt{2}$ compared to most of the literature. This is due to the fact that the matrix $\mathcal{S}^2$ only turns out to be diagonal, if one finally identifies labels which are related by the flip $(l,m,s) \to (k-l, m+k+2, s+2)$.

levels	(h_{21},h_{11})	CY
(1^9)	$(84,0)$	$-$
$(1,1,3,7,43)$	$(67,19)$	$\mathbb{P}_{1,5,9,15,15}[45]$
$(1,1,3,13,13)$	$(103,7)$	$\mathbb{P}_{1,1,3,5,5}[15]$
$(1,1,5,5,19)$	$(65,17)$	$\mathbb{P}_{1,3,3,7,7}[21]$
$(1,1,7,7,7)$	$(112,4)$	$\mathbb{P}_{1,1,1,3,3}[9]$
$(1,3,3,3,13)$	$(75,3)$	$\mathbb{P}_{1,3,3,3,5}[15]$
$(3,3,3,3,3)$	$(101,1)$	$\mathbb{P}_{1,1,1,1,1}[5]$

Table 1: Odd level Gepner models.

As is well known in defining crosscap states one also needs the so-called P-matrix, which is defined as $P = T^{\frac{1}{2}} S\, T^2\, S\, T^{\frac{1}{2}}$ and relates the loop channel and tree channel Möbius amplitudes. For just the $\mathrm{SU}(2)_k$ Kac-Moody algebra it is given by

$$P_{l,l'} = \frac{2}{\sqrt{k+2}}\, \sin \frac{1}{2}(l,l')_k\, \delta^{(2)}_{l+l'+k,0} \tag{2.12}$$

and for the $N=2$ unitary models it reads [24]

$$\begin{aligned} P^{\mathrm{U}(1)_2}_{s_0,s_0'} &= \frac{1}{\sqrt{2}}\, \sigma_{s_0} \sigma_{s_0'}\, e^{-i\pi \frac{s_0 s_0'}{4}}\, \delta^{(2)}_{s_0+s_0',0}\,, \\ \mathcal{P}_{(l,m,s),(l',m',s')} &= \frac{1}{2\sqrt{2k+4}}\, \sigma_{(l,m,s)}\, \sigma'_{(l',m',s')}\, e^{\frac{i\pi}{2}\frac{m\, m'}{k+2}}\, e^{-i\pi \frac{s\, s'}{4}}\, \delta^{(2)}_{s+s',0} \times \\ &\quad \times \left[P_{l,l'}\, \delta^{(2)}_{m+m'+k+2,0} + (-1)^{\frac{l'+m'+s'}{2}}\, e^{i\pi\frac{m+s}{2}}\, P_{l,k-l'}\, \delta^{(2)}_{m+m',0} \right]. \end{aligned} \tag{2.13}$$

The extra sign factors in (2.13) are given by

$$\begin{aligned} \sigma_{s_0} &= (-1)^{h_{s_0} - \Delta_{s_0}} \\ \sigma_{(l,m,s)} &= (-1)^{h^l_{m,s} - \Delta^l_{m,s}} \end{aligned} \tag{2.14}$$

and arise, as roots of the modular T matrix appear in the definition of P. Compared to [24], for the same reason as for the matrix $\mathcal{S}$ we use a different normalization of the matrix $\mathcal{P}$.[1]

Since in the following we restrict ourselves to the case of all levels being odd, we present in table 1 all Gepner models of this type and their corresponding Calabi-Yau manifold. Apparently, for all levels odd the number of tensor factors is either five or nine. Therefore the formulas to be presented in the following sections are derived under the assumption of $r=5,9$ and all levels k_j odd.

3. The orientifold projection

In this section we will consider orientifolds of the original Gepner models. Before discussing the Klein bottle amplitude, we would like to make some general comments about possible orientifold models.

[1]Note, that for s_0 even one always has $\sigma_{s_0}=1$, which is one of the reasons why the NS sector Möbius amplitude is easier to compute in a general way without caring about an extensive amount of extra signs.

3.1 Different types of orientifolds

The partition function (2.9) is the so called diagonal invariant in the sense that it combines left and right moving states with the same U(1) charge. Geometrically, the resulting model describes the type-IIA/B string compactified on a Calabi-Yau space M. For instance, for the $(3)^5$ Gepner model the corresponding Calabi-Yau is just the quintic hypersurface in $\mathbb{CP}_4$. Besides the diagonal partition function Z_D there also exists the charge conjugated partition function Z_C where one combines left and right moving states with opposite U(1) charges in each tensor factor. As is well known this describes the type-IIA/B string on the mirror manifold W. Moreover, under mirror symmetry type-IIB with the diagonal invariant is mapped to type-IIA with the charge conjugated invariant and vice versa.

These relations continue to hold if we perform orientifold constructions, which break the space-time supersymmetry down to $N = 1$. In an orientifold one divides out by the world-sheet parity transformation Ω.[2] Here the Ω orientifold of type-IIB on $Z_{D/C}$ is mapped to the $\Omega\overline{\sigma}$ orientifold of type-IIA on $Z_{C/D}$, where $\overline{\sigma}$ denotes U(1) charge conjugation in each tensor factor. For reasons which will become clear below we call the type IIB orientifold of Z_D the B-type model and the orientifold of Z_C the A-type model. All these relations are summarized in table 2, where entries in the same line are related via mirror symmetry. Due to this relation via mirror symmetry we can restrict ourselves to the discussion of type-IIB orientifold models. Since in type-IIB one has even dimensional orientifold planes and D-branes, one expects that the number of tadpole conditions is related to the number of even cycles in the Calabi-Yau, i.e. it is related to the Hodge number h_{11}. For the pure Gepner models with Z_D generically this number is rather small, whereas for Z_C it is rather big. Therefore, we expect that the A-type orientifold models are much more restrictive than the B-type orientifolds. It is known that by successive orbifolding one can reduce the number of h_{11} while increasing h_{21}. Therefore, for such orbifold models the A-type models might also become more tractable.

	Type-IIB	Type-IIA
proj.	Ω	$\Omega\overline{\sigma}$
B-type	Z_D	Z_C
	M	W
A-type	Z_C	Z_D
	W	M

Table 2: Orientifold models.

For the B-type model it turns out that the resulting tadpoles can be canceled by introducing B-type RS-boundary states, whereas for the A-type model one uses A-type boundary states. After these general comments we are now in the position to compute the Klein bottle amplitude for the B-type orientifold models in detail. The result for the A-type Klein bottle amplitude is briefly stated at the end of the next section.

3.2 The B-type Klein bottle amplitude

The general Klein bottle amplitude is defined as[3]

$$K = \int_0^\infty \frac{dt}{t} \mathrm{Tr}_{cl} \left(\frac{\Omega}{2} e^{-4\pi t \left(L_0 - \frac{c}{24}\right)} \right)$$

[2]There exist more general orientifold models where one combines Ω with some holomorphic involution of the Calabi-Yau. However, such models are not considered in this paper.

[3]See for instance [36] for a review on open string constructions.

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$$= \frac{4}{c}\int_0^\infty \frac{dt}{t^3}\mathrm{Tr'}_{cl}\left(\frac{\Omega}{2}e^{-4\pi t\left(L_0-\frac{c}{24}\right)}\right) \tag{3.1}$$

where $\mathrm{Tr'}_{cl}$ denotes the trace over only the oscillator modes in the closed string sector and the integration over the bosonic zero modes yields the factor $c = (8\pi^2\alpha')^2$. In the following we will set $c = 1$.

Since all states from the range (2.2) couple symmetrically in the diagonal Gepner partition function (B-type) (2.9), they will all appear in the loop channel Klein bottle amplitude. Therefore, this amplitude can easily be written down

$$K^B = 4\int_0^\infty \frac{dt}{t^3}\frac{1}{2^{r+1}}\frac{1}{\eta(2it)^2}\sum_{\lambda,\mu}^{\beta}(-1)^{s_0}\chi_\mu^\lambda(2it)\,. \tag{3.2}$$

The next step is to transform this amplitude into the tree channel by setting $t \to 1/(4l)$ and using a modular S-transformation. The computation is performed using the methods employed in [14] for the computation of the annulus amplitudes. In order to carry out the summation over λ and μ one has to extend the sum $\sum^\beta$ over the entire range (2.2) by introducing δ-functions written as

$$\begin{aligned} \delta_{2\beta_0\bullet\mu\in 2\mathbb{Z}+1} &= \frac{1}{K}\sum_{\nu_0=0}^{K-1} e^{i\pi\nu_0(q_{\mathrm{tot}}-1)}\,, \\ \delta_{\beta_j\bullet\mu\in\mathbb{Z}} &= \frac{1}{2}\sum_{\nu_j=0}^{1} e^{-i\pi\nu_j(s_0+s_j)}\,. \end{aligned} \tag{3.3}$$

After these steps we get

$$\begin{aligned} \widetilde{K}^B &= \int_0^\infty dl\,\frac{8}{2^{3r}\,K}\frac{1}{\prod_j(k_j+2)}\frac{1}{\eta(2il)^2}\sum_{\lambda,\mu}^{ev}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{K-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}(-1)^{\nu_0}\times \\ &\times e^{-i\pi\frac{s_0}{2}\left(s_0'+\nu_0+2\sum\nu_j+2\right)}\prod_{j=1}^{r}\left(\sin(l_j,l_j')_k\; e^{i\pi\frac{m_j}{k_j+2}\left(m_j'+\nu_0\right)}\; e^{-i\pi\frac{s_j}{2}\left(s_j'+\nu_0+2\nu_j\right)}\right)\chi_{\mu'}^{\lambda'}(2il)\,. \end{aligned} \tag{3.4}$$

Taking into account that $l_j + m_j + s_j \in 2\mathbb{Z}$ we can carry out the sum over λ and μ, which gives rise to some delta-functions. After a few manipulations we can write the tree-channel Klein bottle amplitude as

$$\begin{aligned} \widetilde{K}^B &= \int_0^\infty dl\frac{2^5}{2^r\,K}\frac{1}{\eta(2il)^2}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{K-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}(-1)^{\nu_0}\,\delta^{(4)}_{s_0',2+\nu_0+2\sum\nu_j}\times \\ &\times\prod_{j=1}^{r}\left(f(k_j,l_j')\,\delta^{(2k_j+4)}_{m_j',\nu_0}\,\delta^{(4)}_{s_j',\nu_0+2\nu_j}+g(k_j,l_j')\,\delta^{(2k_j+4)}_{m_j',\nu_0+k_j+2}\,\delta^{(4)}_{s_j',\nu_0+2\nu_j+2}\right)\chi_{\mu'}^{\lambda'}(2il) \end{aligned} \tag{3.5}$$

where we have introduced the notation

$$\begin{aligned} f(k,l') &= \sum_{l=0}^{k}\sin(l,l')_k = \tan\left[\frac{\pi}{2}\frac{(l'+1)(k+1)}{k+2}\right]\,\delta^{(2)}_{l',0} = \sqrt{\frac{k+2}{2}}\frac{P^2_{l',k}}{S_{l',0}}\,, \\ g(k,l') &= \sum_{l=0}^{k}(-1)^l\sin(l,l')_k = f(k,k-l') = \sqrt{\frac{k+2}{2}}\frac{P^2_{l',0}}{S_{l',0}}\,. \end{aligned} \tag{3.6}$$

The tree channel Klein bottle amplitude is also given by the overlap of the so far unknown crosscap state $|C\rangle_B$

$$\widetilde{K}^B = \int_0^\infty dl \langle C| e^{-2\pi l H_{cl}} |C\rangle_B\,. \tag{3.7}$$

In order to extract this crosscap state (at least up to sign factors) from the Klein bottle amplitude it is useful to rewrite (3.5) in terms of S and P matrices as

$$\begin{aligned}\widetilde{K}^B &= \frac{2^5 \prod_j \sqrt{k_j+2}}{2^{\frac{3r}{2}} K} \int_0^\infty dl\, \frac{1}{\eta(2il)^2} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0}\, \delta^{(4)}_{s'_0,2+\nu_0+2\sum\nu_j} \times \\ &\times \prod_{j=1}^r \left(\frac{P^2_{l'_j,\epsilon_j k_j}}{S_{l'_j,0}}\, \delta^{(2k_j+4)}_{m'_j,\nu_0+(1-\epsilon_j)(k_j+2)}\, \delta^{(4)}_{s'_j,\nu_0+2\nu_j+2(1-\epsilon_j)} \right) \chi^{\lambda'}_{\mu'}(2il)\,. \end{aligned} \tag{3.8}$$

It can be checked that even though the sum is over the entire range (2.2), the various δ-functions in (3.8) enforce that the GSO projection and the $\beta_j \bullet \mu \in \mathbb{Z}$ conditions are satisfied. From the final expression (3.8) up to possible signs, one can read off the form of the crosscap state

$$\begin{aligned}|C\rangle_B &= \frac{1}{\kappa_c} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \Xi(\lambda',\mu',\nu_0,\nu_j,\epsilon_j)\, \delta^{(4)}_{s'_0,2+\nu_0+2\sum\nu_j} \times \\ &\times \prod_{j=1}^r \left(\frac{P_{l'_j,\epsilon_j k_j}}{\sqrt{S_{l'_j,0}}}\, \delta^{(2k_j+4)}_{m'_j,\nu_0+(1-\epsilon_j)(k_j+2)}\, \delta^{(4)}_{s'_j,\nu_0+2\nu_j+2(1-\epsilon_j)} \right) |\lambda',\mu'\rangle\rangle_c\,. \end{aligned} \tag{3.9}$$

where we have suppressed the contribution from the two uncompactified world-sheet bosons and where $|\lambda',\mu'\rangle\rangle_c$ denotes in the usual way the crosscap Ishibashi states [37]. For the overlap of two such Ishibashi states we choose in each tensor factor

$$\langle\langle \tilde{l},\tilde{m},\tilde{s}| e^{-2\pi l(L_0+\overline{L}_0-\frac{c}{12})} |l,m,s\rangle\rangle_c = \delta_{l,\tilde{l}}\, \delta_{m,\tilde{m}}\, \delta_{s,\tilde{s}}\, \chi^l_{m,s}(2il)\,. \tag{3.10}$$

From (3.8) one can read off the normalization of the crosscap states

$$\frac{1}{\kappa_c^2} = \frac{2^5 \prod_{j=1}^r \sqrt{k_j+2}}{2^{\frac{3r}{2}} K}\,. \tag{3.11}$$

In section 5 we will determine the so far unknown sign factors $\Xi(\lambda',\mu',\nu_0,\nu_j,\epsilon_j)$ from the consistency of the Möbius strip amplitude.

A closer look at (3.8) or (3.9) reveals that only states are allowed to appear, which couple to their charge conjugate in the diagonal torus partition function Z_D. This is very reminiscent to the B-type boundary states introduced by Recknagel/Schomerus. This was the reason why we called these orientifolds of the diagonal invariant B-type orientifolds.

3.3 The A-type Klein bottle amplitude

The computation for the charge conjugated modular invariant Z_C (A-type) is analogous, so that we can keep its presentation short. In the loop channel only states which couple diagonally in Z_C contribute to the trace. Apparently, these are the same states which couple to their charge conjugates in Z_D. As shown in [14], these states satisfy

$$m_j = b \bmod (k_j+2) \tag{3.12}$$

for all j and $b = 0, \ldots, \frac{K}{2}-1$. Therefore, we can write the A-type Klein bottle amplitude as

$$K^A = 4 \int_0^\infty \frac{dt}{t^3} \frac{1}{2^{r+1}} \frac{1}{\eta(2it)^2} \sum_{\lambda,\mu}^{\beta} \sum_{b=0}^{\frac{K}{2}-1} \delta^{(k_j+2)}_{m_j,b}\, (-1)^{s_0}\, \chi^\lambda_\mu(2it)\,. \tag{3.13}$$

Performing the same steps as for the B-type orientifold, one can transform this expression into the tree channel

$$\begin{aligned}
\widetilde{K}^A = & \frac{2^4}{2^{\frac{3r}{2}}} \frac{1}{\prod_j \sqrt{k_j+2}} \int_0^\infty dl\, \frac{1}{\eta(2il)^2} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0}\, \delta^{(4)}_{s'_0, 2+\nu_0+2\sum \nu_j} \times \\
& \times \delta^{(K')}_{\sum_j \frac{K'}{2k_j+4}\left(m'_j+\nu_0+(1-\epsilon_j)(k_j+2)\right)} \prod_{j=1}^{r} \times \\
& \times \left(\frac{P^2_{l'_j,\epsilon_j k_j}}{S_{l'_j,0}}\, \delta^{(2)}_{m'_j+\nu_0+(1-\epsilon_j)(k_j+2),0}\, \delta^{(4)}_{s'_j+\nu_0+2\nu_j+2(1-\epsilon_j),0} \right) \chi^{\lambda'}_{\mu'}(2il)\,.
\end{aligned} \tag{3.14}$$

Completely analogous to (3.9) this can be written as the overlap of a crosscap state with normalization

$$\frac{1}{\kappa_c^2} = \frac{2^4}{2^{\frac{3r}{2}}} \frac{1}{\prod_{j=1}^r \sqrt{k_j+2}}\,. \tag{3.15}$$

The δ functions in (3.14) do not pose any severe constraints, so that essentially all states are allowed to contribute. Therefore, this A-type crosscap state couples to the same closed string modes as the A-type RS-boundary states.

4. RS boundary states

Due to the general philosophy in orientifold model building, one now has to introduce appropriate D-branes to cancel the R-R tadpoles from the Klein bottle amplitude.

4.1 The B-type annulus amplitudes

Since only states which couple to their charge conjugates in Z_D are present in the B-type tree channel Klein bottle amplitude, it is clear that the suitable boundary states to look at are the so-called B-type RS boundary states. This means that these coherent states only contain Ishibashi states satisfying $q_i = -\overline{q}_i$ and $h_i = \overline{h}_i$ in each tensor factor. Therefore, precisely those μ are allowed which satisfy

$$m_j = b \bmod (k_j+2) \tag{4.1}$$

for some $b = 0, \ldots, \frac{K}{2}-1$ and all j. The complete B-type boundary state is given by

$$\begin{aligned}
|\alpha\rangle_B &= |S_0; (L_j, M_j, S_j)_{j=1}^r\rangle_B \\
&= \frac{1}{\kappa^B_\alpha} \sum_{\lambda',\mu'}^{\beta,b} (-1)^{\frac{(s'_0)^2}{2}}\, e^{-i\pi \frac{s'_0 S_0}{2}} \prod_{j=1}^{r} \frac{S_{l'_j,L_j}}{\sqrt{S_{l'_j,0}}}\, e^{i\pi \frac{m'_j M_j}{k_j+2}}\, e^{-i\pi \frac{s'_j S_j}{2}} |\lambda',\mu'\rangle\rangle\,,
\end{aligned} \tag{4.2}$$

where the sum is restricted to those GSO invariant states satisfying also (4.1). It was shown in [14], that computing the overlap between two boundary states of this type and transforming the resulting tree channel annulus amplitude to loop channel via the S-transformation $l \to \frac{1}{2t}$ yields the following loop channel amplitude

$$\begin{aligned} A^B_{\alpha\tilde{\alpha}} &= \frac{N_\alpha\, N_{\tilde{\alpha}}}{2^{\frac{r}{2}+1}\prod_j \sqrt{k_j+2}} \frac{1}{\kappa^B_\alpha \kappa^B_{\tilde{\alpha}}} \int_0^\infty \frac{dt}{t^3}\, \frac{1}{\eta(it)^2} \sum_{\lambda,\mu}^{ev} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0} \times \\ &\quad \times \delta^{(4)}_{s_0,2+\tilde{S}_0-S_0-\nu_0-2\sum\nu_j}\; \delta^{(K')}_{\sum_j \frac{K'}{2k_j+4}\left(m_j+M_j-\tilde{M}_j+\nu_0+\epsilon_j(k_j+2)\right)} \times \\ &\quad \times \prod_{j=1}^r \left(N^{|\epsilon_j k_j - l_j|}_{L_j,\tilde{L}_j}\; \delta^{(2)}_{m_j+M_j-\tilde{M}_j+\nu_0+\epsilon_j(k_j+2),0}\; \delta^{(4)}_{s_j,\tilde{S}_j-S_j-\nu_0-2\nu_j+2\epsilon_j} \right) \chi^\lambda_\mu(it) \end{aligned} \tag{4.3}$$

with $K' = \mathrm{lcm}(k_j+2)$ and where we have also introduced the Chan-Paton factors N_α. The fusion matrix for the SU(2) Kac-Moody algebra is defined by the Verlinde formula

$$N^k_{i,j} = \sum_l \frac{S_{i,l}\, S_{j,l}\, S^*_{l,k}}{S_{0,l}}\,. \tag{4.4}$$

Note, that in contrast to [14] we have written the annulus amplitude in a manifest reflection symmetric way. The condition that in $A^B_{\alpha\alpha}$ the ground state appears with a factor 1/2 (due to the $(1+\Omega)/2$ projector) fixes the normalization factor to be

$$\frac{1}{\left(\kappa^B_\alpha\right)^2} = \frac{2}{2^{\frac{r}{2}} K} \prod_{j=1}^r \sqrt{k_j+2}\,. \tag{4.5}$$

4.2 The A-type annulus amplitudes

For the A-type boundary states only those Ishibashi states coupling in each tensor to their charge conjugates in Z_C are present. These are the same states which couple diagonally in Z_D satisfying $q_i = \overline{q}_i$ and $h_i = \overline{h}_i$ in each tensor factor. Therefore, the boundary state to be introduced to cancel the Klein bottle tadpole are the so called A-type RS-boundary states

$$\begin{aligned} |\alpha\rangle_A &= |S_0; (L_j, M_j, S_j)_{j=1}^r\rangle_A \\ &= \frac{1}{\kappa^A_\alpha} \sum_{\lambda',\mu'}^{\beta} (-1)^{\frac{s_0^2}{2}}\, e^{-i\pi\frac{s'_0 S_0}{2}} \prod_{j=1}^r \frac{S_{l'_j,L_j}}{\sqrt{S_{l'_j,0}}}\, e^{i\pi\frac{m'_j M_j}{k_j+2}}\, e^{-i\pi\frac{s'_j S_j}{2}} |\lambda',\mu'\rangle\rangle\,, \end{aligned} \tag{4.6}$$

which can be straightforwardly transformed into loop channel

$$\begin{aligned} A^A_{\alpha\tilde{\alpha}} &= N_\alpha\, N_{\tilde{\alpha}} \frac{\prod_j \sqrt{k_j+2}}{2^{\frac{r}{2}}\, K} \frac{1}{\kappa^A_\alpha \kappa^A_{\tilde{\alpha}}} \int_0^\infty \frac{dt}{t^3}\, \frac{1}{\eta(it)^2} \sum_{\lambda,\mu}^{ev} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0} \times \\ &\quad \times \delta^{(4)}_{s_0,2+\tilde{S}_0-S_0-\nu_0-2\sum\nu_j} \times \\ &\quad \times \prod_{j=1}^r \left(N^{|\epsilon_j k_j - l_j|}_{L_j,\tilde{L}_j}\; \delta^{(2k_j+4)}_{m_j+M_j-\tilde{M}_j+\nu_0+\epsilon_j(k_j+2),0}\; \delta^{(4)}_{s_j,\tilde{S}_j-S_j-\nu_0-2\nu_j+2\epsilon_j} \right) \chi^\lambda_\mu(it)\,. \end{aligned} \tag{4.7}$$

The normalization factor turns out to be

$$\frac{1}{\left(\kappa_\alpha^A\right)^2} = \frac{K}{2^{\frac{r}{2}+1}} \frac{1}{\prod_{j=1}^r \sqrt{k_j+2}} \,. \tag{4.8}$$

With all the annulus amplitudes available we would like to discuss whether in general we can expect orientifolds of Gepner models to lead to some of the phenomenologically important issues of the Standard Model like unitary gauge symmetries and chirality.

4.3 Supersymmetry and chirality

To begin with let us discuss what kinds of boundary states are generically allowed to be introduced into the orientifold background. Requiring that the annulus amplitudes (4.3) and (4.7) only contain NS-NS and R-R couplings between the different tensor factors implies $S_0 - \tilde{S}_0$ and $S_j - \tilde{S}_j$ even. Making the amplitude (4.3) self-consistent in the sense that only states with $l_j + m_j + s_j = 0$ mod 2 are allowed to contribute also requires $L_j + M_j + S_j = 0$ mod 2 for all j. Moreover, it is evident from the annulus amplitudes that the boundary states also satisfy the reflection symmetry $(L_j, M_j, S_j) \to (k_j - L_j, M_j + k_j + 2, S_j + 2)$ in each tensor factor separately.

As we can see from the annulus amplitudes B-type branes are classified by the combinations

$$M = \sum_j \frac{K' \, M_j}{k_j + 2} \bmod 2K' \,, \qquad S = \sum_j S_j \,. \tag{4.9}$$

As was pointed put in [16], the S_j odd boundary states are sick in the sense, that they do not yield consistent annulus amplitudes together with the S_j even states. Moreover, as we will see in section 6 the crosscap state formally has $\tilde{M}_j = \tilde{S}_j = \tilde{S}_0 = 0$ and therefore is an S_j even state. Due to the reflection symmetry we can always choose $S_j = 0$ for all j, so that we are left with the two distinct possibilities $S_0 \in \{0, 2\}$.

Let us now determine what happens with a RS boundary state $|\alpha\rangle_{A,B}$ under the world-sheet parity transformation Ω. Since $|\alpha\rangle_{B,A}$ contains Ishibashi states satisfying $q_i = -\overline{q}_i$ from the $Z_{D,C}$ partition function, exchanging left and right movers leads to the following action on a boundary state

$$\Omega : |S_0; (L_j, M_j, S_j)_{j=1}^r\rangle_{B,A} \to | - S_0; (L_j, -M_j, -S_j)_{j=1}^r\rangle_{B,A} \,. \tag{4.10}$$

Therefore, only states with $S_0 = 0, 2$, $M_j = 0$ and $S_j = 0, 2$ are actually invariant under Ω. All other states have to be introduced in pairs where Ω maps a boundary state to the charge conjugated one. This means for the spectrum on the branes, that those which are invariant under Ω lead to SO(N) and Sp$(2N)$ gauge groups, whereas all others give rise to U(N) gauge groups. Thus we conclude, that unitary gauge groups are generically present in Gepner model orientifolds.

Now let us discuss the issue of supersymmetry. As in [14] defining the charge

$$Q(\alpha) = -\frac{S_0}{2} - \sum_{j=1}^r \frac{S_j}{2} + \sum_{j=1}^r \frac{M_j}{k_j+2} \,, \tag{4.11}$$

the condition for two such boundary states to preserve the same supersymmetry reads

$$Q(\alpha) - Q(\tilde{\alpha}) = 0 \bmod 2\,. \tag{4.12}$$

Of course in the orientifold setting, in order to get a supersymmetric model the boundary states have also to preserve the same supersymmetry as the crosscap state, which, as we have mentioned, formally has $\tilde{M}_j = \tilde{S}_j = \tilde{S}_0 = 0$. In this paper we are only considering supersymmetric models, so that we really require $Q(\alpha) = 0 \bmod 2$ for all the D-branes we introduce.

Then for B-type boundary states it is obvious that the supersymmetry condition (4.12) with respect to the orientifold plane boils down to $M = 0 \bmod K'$. Therefore, all supersymmetric B-type boundary states are invariant under the Ω projection and carry always orthogonal or symplectic gauge groups. Let us emphasize that for A-type boundary states such a condition does not arise and they can also carry unitary gauge groups.

In view of phenomenological applications, it is also important to know whether the RS boundary states can lead to chiral models. The chiral matter content is geometrically given by the topological (K-theoretic) intersection number between two boundary states. In the CFT this is given by the Witten index, which has been computed in [16]. For B-type boundary states the Witten index reads

$$I_{\alpha,\tilde{\alpha}} = (-1)^{\frac{S-\tilde{S}}{2}} \sum_{m'_j} \delta^{(K')}_{\frac{M-\tilde{M}}{2}+\sum \frac{K'}{2k_j+4}(m'_j+1),0} \prod_{j=1}^{r} N^{m'_j-1}_{L_j,\tilde{L}_j}\,. \tag{4.13}$$

Remember that for supersymmetric D-branes we have $(M - \tilde{M}) = 0 \bmod K'$. Now, inserting (4.12) in (4.13) one immediately realizes that $I(\alpha,\tilde{\alpha}) = I(\tilde{\alpha},\alpha)$. However, one can generically prove that the intersection number is anti-symmetric, leaving only the possibility of vanishing intersection number between two relatively supersymmetric boundary states

$$I_{\alpha,\tilde{\alpha}} = 0\,. \tag{4.14}$$

Thus, we conclude that with just using the highly symmetric B-type RS-boundary states one cannot built chiral supersymmetric orientifolds of Gepner models. This of course does not mean that on Calabi-Yau spaces, there are no chiral intersecting D-brane models, it simply means that the set of RS-boundary states is too restrictive to achieve one of the most salient features of the Standard Model.

For the A-type branes the Witten index is given by

$$I_{\alpha,\tilde{\alpha}} = (-1)^{\frac{S-\tilde{S}}{2}} \sum_{\nu_0=0}^{\frac{K}{2}-1} \prod_{j=1}^{r} N^{2\nu_0+M_j-\tilde{M}_j}_{L_j,\tilde{L}_j} \tag{4.15}$$

and in general it does not vanish for supersymmetric boundary states.[4]

[4] In [8] it was found that for a subset of all supersymmetric A-type branes in the $(3)^5$ Gepner model the Witten index actually vanishes, but we have checked for instance in the $(1,1,7,7,7)$ Gepner model, that this is not a general rule.

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Summarizing, we have found that B-type orientifold models are less constrained due to the smaller number of h_{11}, but that their open string spectrum turns out to be phenomenologically less appealing, as one always gets non-chiral models with orthogonal or symplectic gauge groups. By turning on the massless (anti-)symmetric matter on the branes one can break the gauge symmetry down to unitary gauge factors, however at the cost of actually leaving the RS framework. Of course, even turning on these open string moduli does not lead to chirality. Let us mention that these models are very similar to the orientifolds with D-branes at angles on toroidal orbifolds as discussed in [38]. In these latter models, the way to realize unitary gauge groups and chirality was to move to more general D-brane configurations, which led eventually to the idea of intersecting brane worlds [2]. It would be interesting to figure out whether more general boundary states can be defined for Gepner models, maybe along the way proposed in [18].

On the contrary, for the A-type models unitary gauge groups and chirality are generic features of the open string spectrum. However, here the models become very complex, as there are a large number of tadpole conditions to be satisfied by a very large set of possible boundary states. Whether there really exist non-trivial chiral models with large enough gauge group remains to be uncovered and is beyond the scope of this paper [39]. We end this section by stating that these A-type models are very similar to the ordinary orientifold models on toroidal orbifolds as first discussed in [40, 41].

5. The Möbius strip amplitude

The only unoriented one-loop diagram which remains to be computed is the Möbius strip amplitude

$$M_\alpha = \int_0^\infty \frac{dt}{t} \mathrm{Tr}_{\alpha\alpha'} \left(\frac{\Omega}{2} e^{-2\pi t \left(L_0 - \frac{c}{24}\right)} \right) = \frac{1}{c} \int_0^\infty \frac{dt}{t^3} \mathrm{Tr}'_{\alpha\alpha'} \left(\frac{\Omega}{2} e^{-2\pi t \left(L_0 - \frac{c}{24}\right)} \right) \tag{5.1}$$

where the traces are over open strings stretching between the brane α and its Ω image α'. Generically, due to the Ω insertion one gets extra signs in the Möbius amplitude which can be described by the characters with shifted arguments $\chi(it + 1/2)$. It is very convenient to introduce real characters defined as

$$\hat{\chi}\left(it + \frac{1}{2}\right) = e^{-i\pi\left(h - \frac{c}{24}\right)} \chi\left(it + \frac{1}{2}\right) \tag{5.2}$$

and to express both the loop and tree channel Möbius amplitude in terms of them. Formally the tree channel amplitude can be deduced from the boundary and crosscap states as

$$\widetilde{M}_\alpha = \int_0^\infty dl \langle C| e^{-2\pi l H_{cl}} |\alpha\rangle = \sum_j \Gamma_j\, B_j\, \hat{\chi}_j \left(2il + \frac{1}{2}\right). \tag{5.3}$$

where Γ_j and B_j are the crosscap respectively boundary state coefficients. In order to transform this to the loop channel one applies the P-transformation with $P = T^{\frac{1}{2}} S T^2 S T^{\frac{1}{2}}$. Clearly in order to determine the tree channel Möbius amplitude we have to know the complete crosscap state. In our case, so far we have only fixed the crosscap state up to those extra signs $\Xi(\lambda', \mu', \nu_0, \nu_j, \epsilon_j)$ which we could not detect in the Klein bottle amplitude. Our strategy to find them, is to impose the condition that GSO orbits of hatted characters transform into GSO orbits of hatted characters under the P-transformation.

5.1 P-transformation of orbits

In this section we compute the P-transformation of a GSO orbit, where we are taking very carefully the extra signs into account. For our purposes it is sufficient to just consider the NS sector amplitudes, which under the P-transformation transform among themselves and, as it turns out, are easier to handle in a general way. Assume that in some loop channel Möbius amplitude the following GSO invariant orbit appears

$$M^\lambda_\mu = \sum_{\nu_0=0}^{\frac{K}{2}-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} (-1)^{\left[h^\lambda_\mu(\nu_0,\nu_j)-h^\lambda_\mu\right]} \, \widehat{\chi}^\lambda_{\mu+2\nu_0\beta_0+\sum\nu_j\beta_j}\left(it+\frac{1}{2}\right), \tag{5.4}$$

where $h^\lambda_\mu(\nu_0,\nu_j)$ denote the conformal dimensions of the states appearing in the orbit and $h^\lambda_\mu = h^\lambda_\mu(0,\vec{0})$. These extra signs appear in the amplitude after writing it in terms of the hatted characters. Now we would like to figure out what the resulting amplitude in the tree channel is by applying a P-transformation. The appropriate P-matrix (2.13) introduces a sign factor

$$\prod_{j=1}^{r} \sigma_{(l,m,s)} = (-1)^{\left[h^\lambda_\mu(\nu_0,\nu_j)-\Delta^\lambda_\mu(\nu_0,\nu_j)\right]} \tag{5.5}$$

which combines with the sign in (5.4) in just the right way to cancel the $(-1)^h$ factor and just leaves $(-1)^\Delta$. The former sign is much harder to compute than the latter one, as it requires appropriate reflections into the standard range (2.2). However for the sign $(-1)^\Delta$ we can just use the general formula (2.3) and get the right result. After some little algebra we find

$$\Delta^\lambda_\mu(\nu_0,\nu_j) - h^\lambda_\mu = \nu_0 + \sum_{k<l}\nu_k\nu_l + \sum_j \nu_j\left(\frac{s_0+s_j}{2}+1\right) \mod 2\,. \tag{5.6}$$

The non-trivial quadratic piece $\sum_{k<l}\nu_k\nu_l$ in (5.6) turns out to be quite important for yielding indeed sums over orbits in tree channel. In order to carry out the sums over the ν_j variables in (5.4) one has to evaluate expressions like

$$F_r(\eta_1,\ldots,\eta_r) = \sum_{\nu_1,\ldots,\nu_r=0}^{1}\left(\prod_{k<l}(-1)^{\nu_k\nu_l}\right) e^{i\pi(\nu_1\eta_1+\cdots+\nu_r\eta_r)}, \tag{5.7}$$

which are slightly more complicated than what one is used to from the computation of the Klein bottle and annulus diagrams. By performing the two sums over ν_r and ν_{r-1} explicitly, one derives the following recursion relation

$$F_r(\eta_1,\ldots,\eta_r) = 2\,(-1)^{\eta_{r-1}\eta_r}\, F_{r-2}(\eta_1+\eta_{r-1}+\eta_r+1,\ldots,\eta_{r-2}+\eta_{r-1}+\eta_r+1)\,. \tag{5.8}$$

Iteratively evaluating this relation, some inspection reveals that the final answer for F_r can be written as

$$F_r(\eta_1,\ldots,\eta_r) = \begin{cases} (-1)^s\, 2^{\frac{r}{2}}\, \prod_{k<l}(-1)^{\eta_k\eta_l} \prod_j (-1)^{\eta_j} & \text{for } r=4s \\ (-1)^s\, 2^{\frac{r+1}{2}}\, \prod_{k<l}(-1)^{\eta_k\eta_l}\, \delta^{(2)}_{\sum_j \eta_j,0} & \text{for } r=4s+1 \\ (-1)^s\, 2^{\frac{r}{2}}\, \prod_{k<l}(-1)^{\eta_k\eta_l} & \text{for } r=4s+2 \\ (-1)^s\, 2^{\frac{r+1}{2}}\, \prod_{k<l}(-1)^{\eta_k\eta_l}\, \delta^{(2)}_{\sum_j \eta_j,1} & \text{for } r=4s+3 \end{cases} \tag{5.9}$$

with $s \in \mathbb{Z}_0^+$. Since in our case $r=5$ or $r=9$, the second line in (5.9) is relevant. Note, that the quadratic piece in (5.6) stays form invariant under this discretized version of a Fourier transform. In this sense it is analogous to the invariance of a gaussian under a continuous Fourier transformation. The form invariance of the quadratic piece is very important for the tree and loop channel Möbius amplitudes to contain the correct relative signs in front of the different contributions in the GSO orbits.

Using (5.9) one finds after a few steps the following P-transformed amplitude

$$\begin{aligned} \widetilde{M}^{\lambda}_{\mu} \sim \sum_{\lambda',\mu'}^{\beta} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \left(\prod_{j=1}^{r} \sigma_{(l'_j,m'_j,s'_j)} \right) \left(\prod_{k<l} (-1)^{\eta_k\eta_l} \right) \delta^{(2)}_{\sum_j \eta_j,0}\, e^{-i\pi\frac{s_0\, s'_0}{4}}\, \delta^{(2)}_{s_0+s'_0,0} \times \\ \times \prod_{j=1}^{r} \Bigg(P_{l_j,|\epsilon_j k_j - l'_j|}\, e^{i\pi\frac{m_j\, m'_j}{2k_j+4}}\, \delta^{(2)}_{m_j+m'_j+(1-\epsilon_j)(k_j+2),0}\, e^{-i\pi\frac{s_j\, s'_j}{4}}\, \delta^{(2)}_{s_j+s'_j,0} \times \\ \times (-1)^{\epsilon_j\frac{(m_j+s_j)}{2}} (-1)^{\epsilon_j\frac{(l'_j+m'_j+s'_j)}{2}} \Bigg) \widehat{\chi}^{\lambda'}_{\mu'} \left(2il+\frac{1}{2}\right) \end{aligned} \tag{5.10}$$

with

$$\eta_j = \frac{s_0+s_j}{2} - \frac{s'_0+s'_j}{2} + \epsilon_j + 1\,. \tag{5.11}$$

A little contemplation about this expression and checking it for some concrete examples reveals that indeed it contains again orbits over GSO invariant states with just the right sign in front of each hatted character. After this little exercise, we can now come back to our orientifold models.

5.2 The B-type Möbius amplitude

Apparently the tree channel Möbius strip amplitude (5.10) has a very similar form than our tree channel Klein bottle amplitude. Therefore, we choose (5.10) as the guiding principle to fix the signs in the crosscap state. Since we are eventually only considering supersymmetric models, it is sufficient to consider only the NS-part of the crosscap states. Computing the η_j in (5.11) for the restricted values of the Gepner model labels from (3.9) we find

$$\eta_j = \sum_{i\neq j} \nu_i\,. \tag{5.12}$$

Now taking the signs from (5.10) we finally get

$$|C\rangle^{NS}_B = \frac{1}{\kappa_c} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{\frac{K}{2}-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0} \left(\prod_{k<l} (-1)^{\nu_k\nu_l} \right) (-1)^{\sum_j \nu_j}\, \delta^{(4)}_{s'_0,2+2\nu_0+2\sum\nu_j} \times$$

$$\times \prod_{j=1}^{r} \left(\sigma_{(l'_j,m'_j,s'_j)} \frac{P_{l'_j,\epsilon_j k_j}}{\sqrt{S_{l'_j,0}}} \delta^{(2k_j+4)}_{m'_j,2\nu_0+(1-\epsilon_j)(k_j+2)} \times \right.$$
$$\left. \times\, \delta^{(4)}_{s'_j,2\nu_0+2\nu_j+2(1-\epsilon_j)} (-1)^{\epsilon_j \frac{(m'_j+s'_j)}{2}} \right) |\lambda',\mu'\rangle\rangle_c \,. \tag{5.13}$$

The next logical step is to compute the tree-channel Möbius amplitude between a boundary state and the crosscap state (5.13). Since each Ishibashi state appearing in the crosscap state also appears in the B-type boundary states, the tree-channel Möbius amplitude turns out to be

$$\widetilde{M}^{B,NS}_{\alpha} = -\frac{2N_\alpha}{\kappa_c \kappa_\alpha} \int_0^\infty dl \, \frac{1}{\eta(2il+\frac{1}{2})^2} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{\frac{K}{2}-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0} \left(\prod_{k<l} (-1)^{\nu_k \nu_l} \right) \times$$
$$\times (-1)^{\sum_j \nu_j} e^{i\pi \frac{s'_0 S_0}{2}} \delta^{(4)}_{s'_0,2+2\nu_0+2\sum \nu_j} \times$$
$$\times \prod_{j=1}^{r} \left(\sigma_{(l'_j,m'_j,s'_j)} \frac{P_{l'_j,\epsilon_j k_j} S_{l'_j,L_j}}{S_{l'_j,0}} \delta^{(2k_j+4)}_{m'_j,2\nu_0+(1-\epsilon_j)(k_j+2)} \delta^{(4)}_{s'_j,2\nu_0+2\nu_j+2(1-\epsilon_j)} \times \right.$$
$$\left. \times\, (-1)^{\epsilon_j \frac{(m'_j+s'_j)}{2}} e^{-i\pi \frac{m'_j M_j}{k_j+2}} e^{i\pi \frac{s'_j S_j}{2}} \right) \widehat{\chi}^{\lambda'}_{\mu'} \left(2il + \frac{1}{2} \right), \tag{5.14}$$

where the overall sign has been fixed a posteriori by tadpole cancellation. Using the P-matrix with $l \to 1/8t$ this amplitude can be transformed into loop channel. After quite some algebra using for instance the formula

$$Y^{k-l'}_{L,k-l} = (-1)^{\frac{2L+l-l'}{2}} Y^{l'}_{L,l} \,, \tag{5.15}$$

with the Y tenso r defined below, we finally arrive at the following expression

$$M^{B,NS}_{\alpha} = N_\alpha \frac{(-1)^s}{2^{r+1}} \int_0^\infty \frac{dt}{t^3} \frac{1}{\eta(it+\frac{1}{2})^2} \sum_{\lambda,\mu}^{ev} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \left(\prod_{k<l} (-1)^{\rho_k \rho_l} \right) \delta^{(2)}_{\sum_j \rho_j,0} \, \delta^{(2)}_{s_0,0} \times$$
$$\times \delta^{(K')}_{\sum \frac{K'}{2k_j+4}[2M_j - m_j - \epsilon_j(k_j+2)],0} \times$$
$$\times \prod_{j=1}^{r} \left(\sigma_{(l_j,m_j,s_j)} Y^{l_j}_{L_j,\epsilon_j k_j} \, \delta^{(2)}_{m_j+\epsilon_j(k_j+2),0} \delta^{(2)}_{s_j,0} \, (-1)^{\frac{\epsilon_j}{2}[2S_j - s_j - 2\epsilon_j]} \times \right.$$
$$\left. \times\, (-1)^{\frac{(1-\epsilon_j)}{2}[2M_j - m_j - \epsilon_j(k_j+2)]} \right) \widehat{\chi}^{\lambda}_{\mu} \left(it + \frac{1}{2} \right) \tag{5.16}$$

with $r = 4s+1$ and

$$\rho_j = \frac{s_0 + s_j}{2} + \epsilon_j - 1 \,. \tag{5.17}$$

In (5.16) the integer valued $Y^{l_3}_{l_1,l_2}$ tensor of $\mathrm{SU}(2)_k$ is defined as

$$Y^{l_3}_{l_1,l_2} = \sum_{l=0}^{k} \frac{S_{l_1,l} P_{l_2,l} P_{l_3,l}}{S_{0,l}} \,. \tag{5.18}$$

Note, that the sign factor $(-1)^s$ gives rise to different projections depending on the number of tensor factors. For $r=5$ one always gets orthogonal gauge groups, whereas for the $(1)^9$ Gepner model one finds symplectic gauge groups. Evaluating the general formula for the Möbius amplitude we can now revisit from a formal point of view the question about the kinds of gauge groups living on the RS-boundary states. Recall that relatively supersymmetric D-branes must satisfy $M=0 \bmod K'$. Inserting this into (5.16) one realizes from the $\delta^{(K')}$ constraint that the ground state $(2)(0,0,0)^5$ always appears. Therefore, we can confirm our general expectation that for all supersymmetric D-branes the gauge sector gets symmetrized or anti-symmetrized and no unitary gauge groups are possible.

5.3 The A-type Möbius amplitude

For the A-type crosscap and boundary states the overlap is

$$\begin{aligned}\widetilde{M}_{\alpha}^{A,NS} =& -\frac{2N_\alpha}{\kappa_c^A \kappa_\alpha^A} \int_0^\infty dl\, \frac{1}{\eta(2il+\frac{1}{2})^2} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{\frac{K}{2}-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} (-1)^{\nu_0} \left(\prod_{k<l} (-1)^{\nu_k \nu_l}\right) \times \\ & \times (-1)^{\sum_j \nu_j}\, e^{i\pi \frac{s_0' S_0}{2}}\, \delta^{(4)}_{s_0', 2+2\nu_0+2\sum \nu_j}\, \delta^{(K')}_{\sum_j \frac{K'}{2k_j+4}\left(m_j'+2\nu_0+(1-\epsilon_j)(k_j+2)\right)} \times \\ & \times \prod_{j=1}^{r} \left(\sigma_{(l_j',m_j',s_j')}\, \frac{P_{l_j',\epsilon_j k_j}\, S_{l_j',L_j}}{S_{l_j',0}}\, \delta^{(2)}_{m_j', 2\nu_0+(1-\epsilon_j)(k_j+2)} \times \right. \\ & \left. \times\, \delta^{(4)}_{s_j', 2\nu_0+2\nu_j+2(1-\epsilon_j)}\, (-1)^{\epsilon_j \frac{(m_j'+s_j')}{2}}\, e^{-i\pi \frac{m_j' M_j}{k_j+2}}\, e^{i\pi \frac{s_j' S_j}{2}} \right) \widehat{\chi}^{\lambda'}_{\mu'}\left(2il+\frac{1}{2}\right).\end{aligned} \tag{5.19}$$

The transformation into loop channel yields

$$\begin{aligned}M_{\alpha}^{A,NS} =& N_\alpha \frac{(-1)^s}{2^{r+1}} \int_0^\infty \frac{dt}{t^3}\, \frac{1}{\eta(it+\frac{1}{2})^2} \sum_{\lambda,\mu}^{ev} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \left(\prod_{k<l} (-1)^{\rho_k \rho_l}\right) \delta^{(2)}_{\sum_j \rho_j,0}\, \delta^{(2)}_{s_0,0} \times \\ & \times \prod_{j=1}^{r} \left(\sigma_{(l_j,m_j,s_j)}\, Y^{l_j}_{L_j,\epsilon_j k_j}\, \delta^{(2k_j+4)}_{2M_j-m_j-\epsilon_j(k_j+2),0}\, \delta^{(2)}_{s_j,0}\, (-1)^{\frac{\epsilon_j}{2}[2S_j-s_j-2\epsilon_j]} \times \right. \\ & \left. \times\, (-1)^{\frac{(1-\epsilon_j)}{2}[2M_j-m_j-\epsilon_j(k_j+2)]} \right) \widehat{\chi}^{\lambda}_{\mu}\left(it+\frac{1}{2}\right).\end{aligned} \tag{5.20}$$

Having all one-loop amplitudes available we can move forward and compute the tadpoles and from the loop channel amplitudes the massless spectra. Note that in contrast to earlier approaches to the construction of orientifolds of Gepner models, we have derived absolutely general formulas for all the relevant one-loop amplitudes, which can be easily installed on a computer.

6. Tadpole cancellation

Massless states propagating between the various combinations of crosscap and boundary states give rise to divergences in the respective one-loop diagram. For a consistent string model one has to require that after adding up all 1-loop diagrams these divergences do cancel.

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6.1 B-type models

For the B-type models only those states (λ,μ) are present in the crosscap and boundary states, which appear in Z_D in combination with their charge conjugate ones $(\lambda,-\mu)$. Therefore, besides the vacuum only some of the chiral-antichiral, (ac), states can lead to tadpoles. This implies that the number of independent tadpole cancellation conditions is at most $N_{ac}+1=h_{11}+1$.

In order to cancel the tadpoles from the orientifold planes, we are introducing stacks of B-type boundary states $|S_0;\prod_j(L_j^a,0,0)\rangle$ with L_j even and CP-factors N_a. By extracting the divergences from the various one-loop amplitudes, we find that, as required, they lead to perfect squares. The contribution from a massless state (λ,μ) is given by the intriguingly simple expression

$$\frac{2}{K}\frac{2^r}{\prod_j \sin(l_j,0)_{k_j}}\left(\sum_{a=1}^{N} N_a \prod_j \sin(l_j,L_j^a)_{k_j} - 4\prod_j \sin\frac{1}{2}(l_j,k_j)_{k_j}\right)^2 = 0\,. \qquad (6.1)$$

Using the field identifications, without loss of generality we have assumed that all m_j are even. Moreover, N denotes the number of stacks of different boundary states and N_a the number on each individual stack.

From the general tadpole cancellation conditions (6.1) it is immediately clear that there always exists a simple solution to these equations namely by choosing one stack of D-branes with

$$L_j=\frac{k_j\mp 1}{2} \qquad (6.2)$$

for all j and $k_j=4n_j\pm 1$. The Chan-Paton factor is just $N_1=4$ and for $r=5$ leads to a gauge group SO(4) and for $r=9$ to Sp(4). The interpretation of this solution is that we have just placed appropriate D-branes right on top of the orientifold plane. From the phenomenological point of view this simple solution is not very interesting for its rank is far to small to accommodate the Standard Model gauge symmetry. As we will show in the next section, the set of equations (6.1) allows more general solutions, which can also have amazingly high rank gauge symmetries.

6.2 A-type models

For A-type models massless tadpoles can arise from states which couple to their charge conjugates in Z_C respectively diagonally in Z_D. Therefore the number of tadpole cancellation conditions is given by h_{21} for the Z_D model. As in the B-type model we introduce A-type boundary states $|S_0;\prod_j(L_j^a,M_j,S_j)\rangle$ with CP-factors N_a. As we have argued, here also non-zero values of M_j and S_j are allowed so that in general we have to introduce also the Ω image brane $|-S_0;\prod_j(L_j^a,-M_j,-S_j)\rangle$. By shifting again the labels such that all m_j are even we obtain for the A-type tadpole cancellation the general formula

$$\left(\sum_{a=1}^{N} 2N_a \cos\left[\pi\frac{s_0 S_0^a}{2}-\pi\sum_j\left(\frac{m_j M_j^a}{k_j+2}-\frac{s_j S_j^a}{2}\right)\right]\prod_j \sin(l_j,L_j^a)_{k_j} - 4\prod_j \sin\frac{1}{2}(l_j,k_j)_{k_j}\right)^2 = 0\,. \qquad (6.3)$$

For Ω invariant boundary states the cos[ ] term vanishes and the overall CP-factor is $2N_a$.

From (6.3) it is clear that there also exists a generic solution corresponding to placing four branes just on top of the orientifold plane

$$L_j = \frac{k_j \mp 1}{2}, \; M_j = S_j = S_0 = 0 \tag{6.4}$$

for all j and $k_j = 4n_j \pm 1$. Here again we get SO(4) gauge group for $r = 5$ and Sp(4) for $r = 9$. It would be interesting to evaluate these tadpole cancellation conditions for non Ω invariant boundary states and see whether non-trivial solutions exist [39].

7. Examples

In this section we discuss a couple of B-type examples in more detail. We focus in this paper on B-type examples, where quite easily non-trivial solutions can be found. For the A-type models we have not yet managed to find any non-trivial solutions to the tadpole cancellation conditions and leave a more thorough analysis for future work. Since it is the easiest non-trivial example let us revisit first the $(3)^5$ Gepner model.

7.1 Orientifolds of $(3)^5$ Gepner model

In order to evaluate the general tadpole conditions (6.1) we first have to determine what kinds of massless tadpoles can occur. For the $(3)^5$ Gepner model only two massless (ac) states $(s_0) \prod_j (l_j, m_j, s_j)$ and their charge conjugates do exist, namely

$$(2)(0,0,0)^5 \quad \text{and } (0)(1,1,0)^5 = (0)(2,-4,2)^5 \,, \tag{7.1}$$

where for the second state we have applied the reflection symmetry to guarantee m_j even. The next step is to decide what kinds of RS-boundary states one wants to introduce in order to cancel the two tadpoles. In principle one is free to introduce any set of B-type boundary states and try to cancel the tadpoles with them. We choose here the set of B-type boundary states with $S_0 = 0$ displayed in table 3. In table 3 we have also given the dimension of the open string moduli space of each boundary state and, utilizing the loop channel Möbius amplitude, we have also indicated how these scalars transform under the SO(N_j) gauge symmetry. Inserting these values in the tadpole cancellation conditions (6.1), one realizes

RS-boundary state $(L_1, L_2, L_3, L_4, L_5)$	degeneracy	CP-factor	moduli $n_{(S,A)}$
$(0,0,0,0,0)$	1	N_0	0
$(0,0,0,0,2)$	5	N_1	$4_{(4,0)}$
$(0,0,0,2,2)$	10	N_2	$11_{(8,3)}$
$(0,0,2,2,2)$	10	N_3	$24_{(15,9)}$
$(0,2,2,2,2)$	5	N_4	$50_{(28,22)}$
$(2,2,2,2,2)$	1	N_5	$101_{(51,50)}$

Table 3: Boundary states.

that they can be written in a compact form as

$$\begin{aligned}\sum_{i=0}^{5} N_i\kappa^i &= 4\kappa^5\\ \sum_{i=0}^{5}(-1)^i\, N_i\kappa^{-i} &= -4\kappa^{-5}\end{aligned}\tag{7.2}$$

with $\kappa = \frac{1+\sqrt{5}}{2}$. Adding these two equations one gets

$$2N_0 + N_1 + 3N_2 + 4N_3 + 7N_4 + 11N_5 = 44\,. \tag{7.3}$$

Eliminating N_5 from (7.2) yields a second equation

$$5N_0 - 3N_1 + 2N_2 - N_3 + N_4 = 0\,. \tag{7.4}$$

As expected $N_5 = 4$ with all other CP-factors vanishing is a solution to these two equations. However, there are many others with even larger rank of the gauge group. The largest rank we can get is sixteen and is given by the choice $N_0 = 12$, $N_1 = 20$ which leads to a gauge group

$$G = \mathrm{SO}(12)\times\mathrm{SO}(20)\,. \tag{7.5}$$

This result has first been derived for a freely acting orbifold of the $(3)^5$ Gepner model in [22] and has later been confirmed for the $(3)^5$ model itself in [28]. It is quite remarkable that such high rank solutions do exist and we will confirm their existence also for the other Gepner models discussed in this section.

Given a solution to the tadpole equations one can evaluate the loop channel annulus and Möbius strip amplitudes and read off the massless spectrum. For the model with maximal gauge symmetry above, one finds additional matter multiplets in the following representations

$$4\times(\mathbf{1},\mathbf{210}) + 4\times(\mathbf{12},\mathbf{20})\,. \tag{7.6}$$

Note, that the boundary state $(0,0,0,0,0)$ is rigid in the sense that it does not have any open string moduli space. This property has turned out to be really hard to realize in toroidal orbifold models. From the phenomenological point of view rigid cycles are very attractive, as they do not lead to additional adjoint (anti-symmetric) matter, which might spoil the nice gauge unification properties of the MSSM [42].

7.2 Orientifolds of $(1)^2\,(7)^3$ Gepner model

As a second example we discuss the $(1)^2\,(7)^3$ Gepner model. Since $h_{11} = 4$ we expect at most five tadpole cancellation conditions. However, a deeper look into the (ac) states of the model reveals that only three of the possible five states really appear in the B-type boundary and crosscap states

$$\begin{aligned}&(2)(0,0,0)^5\\ &(0)(1,1,0)^5 = (0)(0,-2,2)^2\,(6,-8,2)^3\\ &(0)(0,0,0)^2\,(3,3,0)^3 = (0)(0,0,0)^2(4,-6,2)^3\,.\end{aligned}\tag{7.7}$$

RS-boundary state $(L_1, L_2, L_3, L_4, L_5)$	degeneracy	CP-factor	moduli $n_{(S,A)}$
$(0,0,0,0,0)$	1	N_0	0
$(0,0,2,0,0)$	3	N_1	$3_{(2,1)}$
$(0,0,4,0,0)$	3	N_2	$5_{(4,1)}$
$(0,0,6,0,0)$	3	N_3	$2_{(2,0)}$
$(0,0,2,2,0)$	3	N_4	$14_{(10,4)}$
$(0,0,4,2,0)$	6	N_5	$20_{(13,7)}$
$(0,0,6,2,0)$	6	N_6	$8_{(7,1)}$
$(0,0,4,4,0)$	3	N_7	$27_{(16,11)}$
$(0,0,6,4,0)$	6	N_8	$13_{(9,4)}$
$(0,0,6,6,0)$	3	N_9	$4_{(4,0)}$
$(0,0,2,2,2)$	1	N_{10}	$45_{(35,10)}$
$(0,0,4,2,2)$	3	N_{11}	$62_{(40,22)}$
$(0,0,6,2,2)$	3	N_{12}	$29_{(25,4)}$
$(0,0,4,4,2)$	3	N_{13}	$84_{(47,37)}$
$(0,0,6,4,2)$	6	N_{14}	$41_{(28,13)}$
$(0,0,6,6,2)$	3	N_{15}	$18_{(17,1)}$
$(0,0,4,4,4)$	1	N_{16}	$112_{(57,55)}$
$(0,0,6,4,4)$	3	N_{17}	$56_{(33,23)}$
$(0,0,6,6,4)$	3	N_{18}	$26_{(19,7)}$
$(0,0,6,6,6)$	1	N_{19}	$11_{(11,0)}$

Table 4: Boundary states for $(1,1,7,7,7)$ Gepner model.

The other two (ac) states are left-right combinations of the form

$$(0)(1,1,0)(0,0,0)(2,2,0)^3_L \bowtie (0)(0,0,0)(1,-1,0)(2,-2,0)^3_R \tag{7.8}$$

and as we said do not couple to the boundary and crosscap states. Now we are introducing the boundary states listed in table 4. It is quite impressive that the initial three tadpole cancellation conditions with in general irrational coefficients can be linear transformed, so that finally one ends up with three equations with only integer coefficients. After some algebra we can write the tadpole cancellation conditions as

$$\begin{aligned}
32 &= N_0 + N_1 + N_2 + 3N_4 + 2N_5 + N_6 + 3N_7 + 2N_8 + 2N_9 + 6N_{10} + 6N_{11} + \\
&\quad +3N_{12} + 7N_{13} + 5N_{14} + 4N_{15} + 8N_{16} + 5N_{17} + 3N_{18} + N_{19}\,, \\
0 &= 2N_1 - N_2 - N_3 + N_4 - 2N_6 + N_8 + 2N_9 + 3N_{10} - N_{11} - 2N_{12} + N_{13} + \\
&\quad +N_{14} + 3N_{15} - N_{18} - 3N_{19}\,, \\
0 &= -6N_0 - N_2 + 5N_3 - 7N_4 + 4N_5 + 4N_6 - 2N_7 - N_8 - 6N_9 - 3N_{10} + N_{11} + \\
&\quad +8N_{12} + N_{13} - 3N_{14} - 7N_{15} + 2N_{17} + 3N_{18} + 9N_{19}\,.
\end{aligned} \tag{7.9}$$

The trivial solution is here $N_{16} = 4$, but there again exist solutions with larger rank. Choosing for instance $N_0 = 8$ and $N_5 = 12$ with all other CP-factors vanishing yields a gauge group

$$G = \mathrm{SO}(8) \times \mathrm{SO}(12) \tag{7.10}$$

of rank $\mathrm{rk}(G) = 10$. Evaluating the loop channel annulus and Möbius amplitudes, one finds additional matter multiplets in the following representations

$$13 \times (\mathbf{1}, \mathbf{78}) + 7 \times (\mathbf{1}, \mathbf{66}) + 3 \times (\mathbf{8}, \mathbf{12})\,. \tag{7.11}$$

A second solution with even larger rank is given by the choice $N_0 = 8$, $N_3 = 24$ and $N_9 = 12$ leading to a gauge group of rank $\mathrm{rk}(G) = 22(!)$

$$G = \mathrm{SO}(8) \times \mathrm{SO}(24) \times \mathrm{SO}(12)\,. \tag{7.12}$$

The resulting massless matter transforms in the following representations

$$2 \times (\mathbf{1}, \mathbf{300}, \mathbf{1}) + 4 \times (\mathbf{1}, \mathbf{1}, \mathbf{78}) + 2 \times (\mathbf{8}, \mathbf{24}, \mathbf{1}) + 1 \times (\mathbf{8}, \mathbf{1}, \mathbf{12}) + n_{39} \times (\mathbf{1}, \mathbf{24}, \mathbf{12}) \tag{7.13}$$

with $n_{39} = 5$ for $\vec{L}_3 = (0,0,6,0,0)$ and $\vec{L}_9 = (0,0,6,6,0)$ and $n_{39} = 1$ for $\vec{L}_3 = (0,0,6,0,0)$ and $\vec{L}_9 = (0,0,0,6,6)$.

7.3 Orientifolds of $(1)\,(3)^3\,(13)$ Gepner model

The last example we want to discuss is the $(1)\,(3)^3\,(13)$ Gepner model which has $h_{11} = 3$ so that we expect at most four tadpole constraints. Indeed the (ac) states which are coupling to the boundary and crosscap states are

$$\begin{aligned}
&(2)(0,0,0)^5 \\
&(0)(1,1,0)^5 = (0,-2,2)(2,-4,2)^3\,(12,-14,2) \\
&(0)(0,0,0)(1,1,0)^3\,(6,6,0) = (0)(0,0,0)(2,-4,2)^3\,(6,6,0) \\
&(0)(1,1,0)(0,0,0)^3\,(10,10,0) = (0)(0,-2,2)(0,0,0)^3\,(10,10,0)\,.
\end{aligned} \tag{7.14}$$

For this example we are here only considering the boundary states listed in table 5. A more complete list of boundary states and the resulting tadpole cancellation conditions can be

RS-boundary state $(L_1, L_2, L_3, L_4, L_5)$	degeneracy	CP-factor	moduli $n_{(S,A)}$
$(0,0,0,0,0)$	1	N_0	0
$(0,0,0,0,2)$	1	N_1	0
$(0,2,0,0,4)$	3	N_2	$8_{(7,1)}$
$(0,0,0,0,6)$	1	N_3	$7_{(4,3)}$
$(0,2,0,0,6)$	3	N_4	$15_{(9,6)}$
$(0,2,2,0,6)$	3	N_5	$31_{(17,14)}$
$(0,2,2,2,6)$	1	N_6	$63_{(32,31)}$
$(0,2,0,0,10)$	3	N_7	$8_{(7,1)}$
$(0,0,0,0,12)$	1	N_8	0

Table 5: Boundary states for the $(1,3,3,3,13)$ Gepner model.

found in A. For this choice of boundary states, it turns out that only three out of the four tadpole conditions are actually independent

$$\begin{aligned} 72 &= 4N_0 + 5N_1 + 3N_2 + 4N_3 + 7N_4 + 11N_5 + 18N_6 + 8N_7 - N_8 \\ 0 &= N_0 + N_1 - N_2 + N_7 - N_8 \\ 0 &= 5N_1 - N_2 + 2N_3 - N_4 + N_5 + 2N_7 - 3N_8\,. \end{aligned} \tag{7.15}$$

Choosing for instance

$$N_0 = 8, N_1 = 12, N_8 = 20 \tag{7.16}$$

with the remaining CP-factors vanishing leads to the gauge group

$$G = \mathrm{SO}(8) \times \mathrm{SO}(12) \times \mathrm{SO}(20) \tag{7.17}$$

of rank $\mathrm{rk}(G) = 20$. For this model one finds additional matter multiplets in the following representations

$$3 \times (\mathbf{8}, \mathbf{1}, \mathbf{20}) + 4 \times (\mathbf{1}, \mathbf{12}, \mathbf{20})\,. \tag{7.18}$$

We hope that these examples have convinced the reader that orientifolds of Gepner models though complicated can still be constructed in a systematic way and that they admit surprising and new patterns of solutions to the tadpole conditions.

8. Conclusions

In this paper we have developed a systematic approach for constructing orientifolds of Gepner models. Extending previous work, at least for odd levels, we presented quite general expressions for all A-type and B-type one-loop amplitudes. From these amplitudes we derived the general form of the tadpole cancellation conditions, which in the end turned out to be fairly simple. Finally, we have also discussed a couple of B-type examples in some more detail, in particular confirming that solutions of higher rank generically exist. Moreover, we pointed out on general grounds that for relatively supersymmetric B-type boundary states only orthogonal or symplectic gauge groups with necessarily non-chiral massless matter are possible.

This work is meant to be just a first step into an even more general investigation of orientifolds of Gepner models, where one would also include the cases with even levels and just four tensor factors. The hope is, that for these more general cases some of the limitations of the B-type type models might be overcome. It might also be possible that one has to move beyond the RS-boundary states to realize these phenomenologically appealing features in B-type orientifolds of Gepner models.

Moreover, it remains to be seen whether non-trivial examples of A-type orientifold models do exist, where one can indeed get unitary gauge groups and chiral fermions. One possibility in order to reduce the number of conditions is to consider additional orbifolds or simple current constructions [22, 28]. Encouragingly, for a simple current extension of the A-type orientifold of $(1,1,7,7,7)$ Gepner model one example of a higher rank gauge group was found in [22]. From the phenomenological point of view, the final aim of all

$(L_1, L_2, L_3, L_4, L_5)$	degeneracy	CP-factor	moduli $n_{(S,A)}$
$(0,0,0,0,0)$	1	N_0	0
$(0,2,0,0,0)$	3	N_1	$2_{(2,0)}$
$(0,2,2,0,0)$	3	N_2	$5_{(4,1)}$
$(0,2,2,2,0)$	1	N_3	$12_{(9,3)}$
$(0,0,0,0,2)$	1	N_4	0
$(0,2,0,0,2)$	3	N_5	$4_{(4,0)}$
$(0,2,2,0,2)$	3	N_6	$11_{(8,3)}$
$(0,2,2,2,2)$	1	N_7	$24_{(15,9)}$
$(0,0,0,0,4)$	1	N_8	$3_{(3,0)}$
$(0,2,0,0,4)$	3	N_9	$8_{(7,1)}$
$(0,2,2,0,4)$	3	N_{10}	$18_{(13,5)}$
$(0,2,2,2,4)$	1	N_{11}	$37_{(24,13)}$
$(0,0,0,0,6)$	1	N_{12}	$7_{(4,3)}$
$(0,2,0,0,6)$	3	N_{13}	$15_{(9,6)}$
$(0,2,2,0,6)$	3	N_{14}	$31_{(17,14)}$
$(0,2,2,2,6)$	1	N_{15}	$63_{(32,31)}$
$(0,0,0,0,8)$	1	N_{16}	$4_{(4,0)}$
$(0,2,0,0,8)$	3	N_{17}	$11_{(8,3)}$
$(0,2,2,0,8)$	3	N_{18}	$24_{(15,9)}$
$(0,2,2,2,8)$	1	N_{19}	$50_{(28,22)}$
$(0,0,0,0,10)$	1	N_{20}	$3_{(3,0)}$
$(0,2,0,0,10)$	3	N_{21}	$8_{(7,1)}$
$(0,2,2,0,10)$	3	N_{22}	$18_{(13,5)}$
$(0,2,2,2,10)$	1	N_{23}	$37_{(24,13)}$
$(0,0,0,0,12)$	1	N_{24}	0
$(0,2,0,0,12)$	3	N_{25}	$2_{(2,0)}$
$(0,2,2,0,12)$	3	N_{26}	$5_{(4,1)}$
$(0,2,2,2,12)$	1	N_{27}	$12_{(9,3)}$

Table 6: Boundary states for the $(1,3,3,3,13)$ Gepner model.

these efforts would be to systematically analyze such orientifold models for their ability to yield models which come close to the Standard Model. After all, so far we have just revealed the tip of the iceberg of the whole plethora of Gepner model orientifolds.

It would also be interesting to see whether the orientifold models constructed in this paper are dual after compactification on a circle to the SCFTs for G_2 manifolds proposed in [43].

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A. Tadpole conditions for $(1, 3, 3, 3, 13)$ Gepner model

In this appendix we present the general set of boundary states for the B-type orientifold of the $(1, 3, 3, 3, 13)$ Gepner model. Introducing the boundary states listed in table 6 the four tadpole cancellation conditions read

$$\begin{aligned}
72 &= 4N_0 + 2N_1 + 6N_2 + 8N_3 + 5N_4 + 5N_5 + 10N_6 + 15N_7 + 6N_8 + 3N_9 + 9N_{10} + \\
&\quad +12N_{11} + 4N_{12} + 7N_{13} + 11N_{14} + 18N_{15} + 5N_{16} + 10N_{17} + 15N_{18} + 25N_{19} + N_{20} + \\
&\quad +8N_{21} + 9N_{22} + 17N_{23} - N_{24} + 2N_{25} + N_{26} + 3N_{27}\,, \\
0 &= -5N_4 + 3N_5 - 2N_6 + N_7 - 2N_8 + N_9 - N_{10} - 2N_{12} + N_{13} - N_{14} + 3N_{16} - 2N_{17} + \\
&\quad +N_{18} - N_{19} + 3N_{20} - 2N_{21} + N_{22} - N_{23} + 3N_{24} - 2N_{25} + N_{26} - N_{27}\,, \\
0 &= -N_1 - N_2 - 2N_3 - N_5 - N_6 - 2N_7 + N_8 + N_{10} + N_{11} - N_{16} - N_{17} - 2N_{18} - \\
&\quad -3N_{19} - N_{20} - N_{22} - N_{23} + N_{25} + N_{26} + 2N_{27}\,, \\
0 &= -N_0 - N_2 - N_3 - N_4 - N_6 - N_7 - N_8 + N_9 + N_{11} - N_{17} - N_{18} - 2N_{19} + N_{20} - \\
&\quad -N_{21} - N_{23} + N_{24} + N_{26} + N_{27}\,.
\end{aligned} \tag{A.1}$$

The trivial solution with just the D-branes on top of the orientifold plane corresponds to $N_{15} = 4$ with all other CP-factors vanishing.

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